TEMPERATURE FIELD IN STUDDED SCREENS OF VAPOR GENERATOR

A. L. Bychkovskii and A. L. Lubny-Gertsyk

UDC 536.212:621.181.61

By approximating the packing in the vicinity of a stud of radius r_s by a cylindrical layer of radius R the complicated field of temperatures in the studs $t_s(x, r)$ and the packing $t_p(x, r)$ satisfying the Laplace equation can be written in the form of hyperbolic sine functions (sh(Kx)) and Bessel functions of the first (J(Kr)) and second (Y(Kr)) kind. Integrating the given temperature field with respect to the radius r we obtain an expression for the average temperatures through the cross section of the stud $\bar{t}_s(x)$ and the packing $\bar{t}_p(x)$ (in excess of the wall temperature of the tube $\bar{t}_w$):

$$\bar{t}_s(x),\ \bar{t}_p(x) = t_0\left[\left(1-\frac{x-\Delta}{l}\right) \mp \theta\,\frac{C_0 \operatorname{sh} K\frac{l-x}{R} \mp C_1 \operatorname{sh} K\frac{\Delta}{R}}{\operatorname{sh} K\frac{l}{R}}\right]. \tag{1}$$

The – sign pertains to $\bar{t}_s(x)$ when $C_0 = 1$, while the + sign pertains to $\bar{t}_p(x)$.

Here the characteristic value K is determined as the root of the equation

$$\frac{\lambda_p}{\lambda_s} = \frac{J_1(\rho)}{J_0(\rho)} \cdot \frac{J_0(\rho) - \frac{J_1(K)}{Y_1(K)} Y_0(\rho)}{J_1(\rho) - \frac{J_1(K)}{Y_1(K)} Y_1(\rho)};\quad \rho = K\frac{r_s}{R}, \tag{2}$$

where λ_p and λ_s are the heat-conduction coefficients of the packing and stud. The numbers K can also be determined from a nomogram.

The expressions for the average concentrations of heat fluxes at the stud $\bar{K}_s(x)$ and the packing $\bar{K}_p(x)$ and the heat flux q averaged over the entire cross section have an analogous form:

$$\bar{K}_s(x) = \frac{\bar{q}_s(x)}{q} = \frac{\lambda_s}{\lambda_p} \cdot \frac{1-\theta\frac{Kl}{R} \cdot \frac{\operatorname{ch} K\frac{l-x}{R}}{\operatorname{sh} K\frac{l}{R}}}{1+C_1}, \tag{3}$$

$$\bar{K}_p(x) = \frac{\bar{q}_p(x)}{q} = \frac{1 - K_s(x)\left(\frac{r_s}{R}\right)^2}{1-\left(\frac{r_s}{R}\right)^2}, \tag{4}$$

$$q = \frac{\lambda_p(1+C_1)}{l} t_0,\ \mathrm{W/m^2}. \tag{5}$$

In the expressions (1) and (3)-(5) the field form t_0, the ratio of field forms $\theta = t_1/t_0$, and the calculated complexes C have the following values:

$$t_0 = \frac{\bar{t}^0 + \bar{t}_w - t_i}{1 + C_s + C_1\theta};\quad \theta \cong \frac{\lambda_s - \lambda_p(1+C_1)}{\left[\lambda_s\frac{K}{R} + \alpha(1+C_1)\right]l}; \tag{6}$$

$$C_0 = \frac{\lambda_s}{\lambda_p}\frac{r_s^2}{R^2 - r_s^2};\quad C_1 = \left(\frac{r_s}{R}\right)^2\left(\frac{\lambda_s}{\lambda_p} - 1\right); \tag{7}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 1, pp. 152-174, January, 1972. Original article submitted December 11, 1969; abstract submitted March 13, 1971.

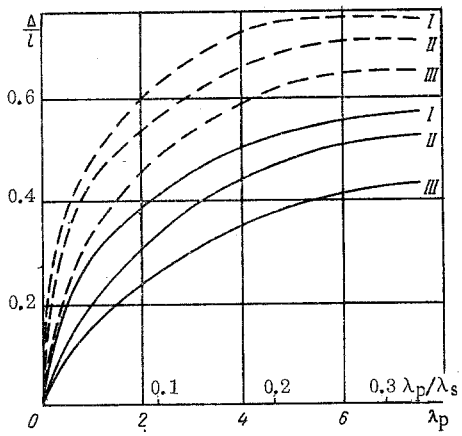

Fig. 1. Relative position of isothermal surface (Δ/l) as a function of the ratio of thermal conduction coefficients of packing and stud (λ_p/λ_s), the radii of the stud and packing (r_s/R), and the height of the stud (H). For r_s/R: I) 0.556; II) 0.455; III) 0.400. Dashed lines: H = 24; solid lines: H = 12.

$$C_s = \frac{\Delta}{l} + \frac{\Delta\bar{t}_w}{t_0} + \theta C_1 C_9;\ C_9 = \frac{\operatorname{sh} K\frac{\Delta}{R}}{\operatorname{sh} K\frac{l}{R}}. \qquad (8)$$

Here $\bar{t}^0$ is the selected average temperature of the studded surface corresponding to the fixed temperature t_f of the flame or to the thermal flux q; α is the coefficient of radiant heat transport, proportional to the cube of the absolute temperature $\bar{t}^0$; t_i is the temperature of the medium flowing through the tube.

Since the expression obtained for finding the distance of the isothermal surface from the base of the tube (Δ) and the top of a stud (l) is somewhat cumbersome we are limited to presenting the relative position of this surface in Fig. 1.

From (1) one can obtain an expression for the average excess temperatures at the characteristic (calculated) points at the top of the stud ($\bar{t}^0_s$) and packing ($\bar{t}^0_p$), the stem of the stud ($\bar{t}^\Delta_s$), and the wall over the packing layer ($\bar{t}^\Delta_p$):

$$\bar{t}^0_s = t_0\left[\left(1+\frac{\Delta}{l}\right)+\theta(C_1C_9-1)\right] \cong C_s K^\Delta_s q\frac{H}{\lambda_s},\ \text{where } 1.02 < C_s < 1.12, \qquad (9)$$

$$\bar{t}^0_p = t_0\left[\left(1+\frac{\Delta}{l}\right)+\theta(C_1C_9+C_0)\right] \cong C_p K^\Delta_p q\frac{H}{\lambda_p},\ \text{where } 1.0 < C_p < 1.25, \qquad (10)$$

$$\bar{t}^\Delta_s = t_1(1+C_1)C_9 \cong 9\cdot10^{-5} q\left(K^\Delta_s - 1{,}1\right), \qquad (11)$$

$$\bar{t}^\Delta_p = \bar{t}^\Delta_s\frac{C_0-C_1}{1+C_1}. \qquad (12)$$

The field of local excess temperatures on the outer wall surface is

$$\bar{t}^\Delta_w(r) = \frac{\bar{t}^\Delta_s J_0\left(3{,}83\frac{r}{R}\right)}{0{,}522\frac{R}{r_s}J_1\left(3{,}83\frac{r_s}{R}\right)}. \qquad (13)$$

The calculation of multiple variants on a computer demonstrated the possibility of describing the concentration of heat fluxes in the stem of a stud (K^Δ_s) by the function $K^\Delta_s = f(\lambda_s/\lambda_p,\ r_s/R,\ H/R)$, the nomogram of which is based on the normative method of calculating the temperatures of the studded heating screens of vapor boilers.

Simplified equations, transformed from the exact expression (1) to the form of a "flat" wall equal in thickness to the stud height H, reduce to the right-hand part of Eqs. (9)-(11). Calculations from the simplified equations lead to results in close agreement with experiment.

*In engineering calculations the experimental values of the contact resistance $1/K_s \cong 1\cdot10^{-3}$ W/m$^2\cdot$°C should be introduced:

$$\bar{t}^0_p = q\left(\frac{1}{K_s} + C_p K^\Delta_p\frac{H}{\lambda_p}\right).$$

É. A. Iodko and E. N. Trynkina

UDC 669.18;51

A solution has been found for the problem of two-phase zone formation during crystallization of solids of extremely simple shape (plates, cylinders, and spheres), taking into account the heating of the melt in the liquid "nucleus."

The problem was solved in the "quasimonophasic" approximation, using the concept of the "solid-crystal fraction" (1) per unit melt volume. The heat and mass transfer equations were supplemented with terms corresponding to the heat and mass sources within the two-phase zone. These sources are proportional to the change in the solid-crystal fraction l with time at a point with the coordinate n. The rule for the change in the interface between the diphasic zone and the solid shell is assumed to be known (and given): $n = \varepsilon(F_0)$. It is determined by the conditions at the outer surface of the casting. The coordinate of the interface between the diphasic zone and the liquid "nucleus," the temperature and concentration fields in the liquid nucleus and diphasic zone, and the solid-crystal fraction l in the diphasic zone were calculated in a computer. It was found that an increase in the initial heating of the melt or the solidification rate produces a decrease in the width of the diphasic zone (Fig. 1). With all other conditions equal, the latter reaches its greatest extent in spherical castings and its smallest in flat castings. This is obviously due to different rates of heat removal from the heated "nucleus" in these two cases. An increase in the initial carbon content of the melt also markedly increases the width of the diphasic zone.

It was also found that the solid-crystal fraction (l), represented as a function of the relative coordinate

$$\nu = \frac{\varepsilon - \eta}{\varepsilon - \xi}$$

is virtually independent of time:

$$l(F_0, \eta) \approx l\left[\frac{\varepsilon(F_0) - \eta}{\varepsilon(F_0) - \xi(F_0)}\right].$$

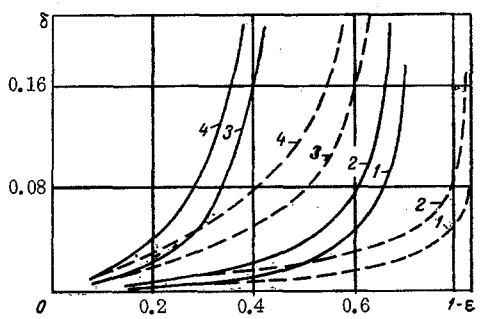

Fig. 1. Width of diphasic zone δ as a function of thickness of solidifying layer $1-\varepsilon$ for initial impurity concentration $C_0 = 0.1$. 1) $K = 1.5$, $t_i = 13.2 \cdot 10^{-4}$; 2) 1.5 and $0.66 \cdot 10^{-4}$ respectively; 3) 0.75 and $13.2 \cdot 10^{-4}$; 4) 0.75 and $0.66 \cdot 10^{-4}$. The solid lines represent a sphere and the dash lines a plate.

Donetsk State University. Original article submitted January 20, 1971; abstract submitted April 20, 1971.

DIFFUSIVE PARTICLE PRECIPITATION IN THE INLET SEGMENT OF A TUBE*

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UDC 533.15

The convective-diffusion equation has been solved for the case of cylindrical symmetry, taking into account the influence of the process by which a parabolic velocity profile is established with a uniform velocity distribution at the input. Since there are no expressions that would be valid over the entire interval, we used formulas obtained for the longitudinal velocity component by solving the Ossian equation and the modified Schiller method. The transverse component of the velocity was found from the medium-compressibility condition.

The problem was programmed for computation in a BÉSM-4 computer. The following values were selected for the Schmidt number: 1) Sc =10; 2) Sc = 2.206; 3) Sc = 0.9.

For air ($\nu = 0.15$ cm^2/sec), this corresponds to diffusion constants D of 0.015, 0.068, and 0.167 cm^2/sec. The results of the calculation showed that even for Sc = 0.9, the "inlet effect" did not exceed ~10%. As applied to aerosol particles suspended in air, this means that the influence of the inlet section is slight.

METHODS UTILIZING TWO TEMPERATURE–TIME INTERVALS†

V. S. Vol'kenshtein

UDC 536.2.083

We will consider a system of solids composed of plane-parallel plates and a semibounded cylinder (heat sink). Different methods utilizing two temperature–time intervals correspond to different compositions for the bodies comprising this system. Two variants of the first method [1] are based on the temperature field of one plate and the heat sink. Two variants of the second method correspond to the temperature field of the three-component system consisting of two plates and the heat sink. The third method is based on the temperature field of a system composed of the heat sink and three plates, two of which are metallic. The existence of interrelated methods makes it possible to investigate solids, liquids, powders, fabrics, and so forth with a single measurement procedure and the same apparatus.

Measurement of thermophysical characteristics by any of the methods in this group reduces to establishment of two time intervals $\Delta\tau_1$ and $\Delta\tau_2$ corresponding to two given changes in the galvanometer readings $\Delta N_1 = N_1 - N_2$ and $\Delta N_2 = N_1 - N_3$ for an instrument connected into the circuit of a differential thermocouple. The thermophysical characteristics measured by any method of this group are calculated from the same formulas: $a = h^2/4p\Delta\tau_1$; $\lambda = b\varepsilon\sqrt{a} = (bh/2L)(\varepsilon h)$, where b and L are two characteristics of the heat sink, found in calibrating it. The dimensionless parameters p and ε or p and (εh) are taken from the working tables, which are compiled beforehand from the appropriate temperature-field equations [2] for fixed values of N_1/N_0, N_2/N_0, and N_3/N_0; h is the thickness of the layer of material under investigation.

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CALCULATION OF TRANSIENT PROCESSES OF VAPOR (GAS) FLOW IN A SYSTEM OF VESSELS

B. A. Arkad'ev, S. A. Dizenko, and V. S. Nemirov

UDC 621.165-501.22

The problem of gas overflow and the filling of gas vessels has a very important practical significance, in particular for calculations of the dynamic variation in the number of rotor revolutions during the sudden closing of tubing valves owing to a drop in the load. The solution is complicated by variability of the hydraulic resistances, by removal from the vapor (gas) of part of the energy, and by other effects.

If the volume of the line joining two vessels is negligibly small the flow of vapor between them at each instant can be found from stationary state equations. It is convenient to consider the flow into the vessel under consideration as positive and flow out of it as negative. In cases when the vapor is moist some moisture can collect in the vessel, part of which evaporates upon a drop in the pressure and must be taken into account in the flow rate balance.

In order to determine the state of the vapor in the vessel it is necessary to know any two parameters, for example the enthalpy and the specific volume, which can be determined as a function of the time on the basis of the laws of mass and energy conservation. In a line joining two vessels the energy at the output can differ from the energy at the input owing to removal (supply) of part of the energy, depending on the variable efficiency coefficient.

On the basis presented systems of equations are obtained for finding the principal parameters for an ideal gas, a superheated vapor, and a moist vapor. Since in all three cases the systems are essentially nonlinear it is expedient to solve them using a computer, although it is feasible to solve them "by hand."

The method worked out was used for the determination of the time variation in the quantity and efficiency of the vapor enclosed within the cavity of the turbine system following the closing of the regulating valves. The loss in revolutions of this turbine calculated from the data obtained is 10.8% (324 rpm) and differs from the result obtained in a turbine test for a drop in the load by 5 rpm, which can serve as indirect confirmation of the acceptability of the proposed method.

A calculation of the loss in revolutions of the same turbine by a method used previously gives 18.2% (546 rpm). Since the permissible loss is limited by the requirements of durability of the rotating components and the adjustment of the automatic safety devices comes to 11-12%, the increase in the calculated value of the loss can serve as a basis for taking constructive measures for its reduction, which leads to an unjustifiable complication and increase in cost of the turbine. Therefore the proposed method allows not only a more exact calculation of the dynamic loss in revolutions but in a number of cases (including the one under consideration) a simplified turbine construction.

Original article submitted October 30, 1969; abstract submitted June 25, 1971.

AXIALLY SYMMETRIC STATIONARY HEAT-CONDUCTION PROBLEM WITH MIXED TEMPERATURE CONDITIONS

V. N. Grebenshchikov

UDC 536.21

The problem is considered of determining the temperature on the half-space boundary z = 0 using the cylindrical coordinates r, φ, z. The half-space is considered to be uniform and isotopic as regards its thermal properties. It is assumed that the thermal characteristics of the material are independent of temperature. At the point (z = h, r = 0) a stationary heat source is located with intensity Q. Convective heat exchange takes place with the surrounding medium within the circle of radius R (in accordance with Newton's law). A specified temperature is maintained outside the circle of radius R. The axially symmetric problem is solved. It is assumed that the temperature function satisfies the heat-conduction equation.

The solution of this mixed problem for a half-space is first reduced to the solving of a pair of integral equations and this in turn is reduced to the solving of a Fredholm integral equation of the second kind for the auxiliary function G(s):

$$G(s)-\frac{\alpha R}{\pi k}\int_0^1 G(t)\ln\left|\frac{s+t}{s-t}\right|dt=F(s),$$

where α and k are the heat-conduction and the heat-exchange coefficients respectively.

The distribution law of temperature inside the circle of radius R is expressed in terms of the auxiliary function G(s) as follows:

$$T(r)=\frac{2R}{\pi}\int_{r/R}^1\frac{G(s)\,ds}{\sqrt{R^2s^2-r^2}}.$$

To solve the integral equation the kernel is replaced by a degenerate one; to this end the kernel is expanded into a series of Chebyshev polynomials,

$$\ln\left|\frac{s+t}{s-t}\right|=4\sum_{n=1}^{\infty}\frac{1}{2n-1}T_{2n-1}(s)\,T_{2n-1}(t).$$

If the right-hand side of the equation can be expanded into a series of Chebyshev polynomials then the solution of the integral equation can also be sought as a series of Chebyshev polynomials,

$$G(s)=\sum_{n=1}^{\infty}G_nT_{2n-1}(s).$$

The coefficients G_n of the series can be determined from the infinite system of algebraic equations

$$G_n+\frac{2\alpha R}{\pi k}\sum_{n=1}^{\infty}G_k\omega_{nk}=f_n\quad(n=1,\ 2,\ 3\ \ldots),$$

where

$$\omega_{nk}=\frac{(2n-1)^2+(2k-1)^2-1}{(n-1/2)[4(n-k)^2-1][4(n+k-1)^2-1]}.$$

The system has a regularity parameter given by

$$1-\frac{8\alpha R}{\pi k}\cdot\frac{2(2n-1)^2-1}{(2n-1)(4n-3)(4n-1)},$$

therefore for $k > 0.85\,\alpha R$ the system is regular, otherwise it is only quasiregular. For an approximate solution the infinite system is cut off.

An example is given to show the calculations involved and also to show that in practice it suffices to retain only three terms of the series expansion of the kernel.

Saratov Polytechnic Institute. Original article submitted July 14, 1969; abstract submitted June 24, 1971.

TEMPERATURE EFFECT OF H_{10} WAVE ON THE ABSORBING WALL OF A RECTANGULAR WAVEGUIDE*

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UDC 621.317.08:621.372.8

In order to measure high and superhigh levels of the transmitted microwave power the method of absorbing walls is the most acceptable since it enables us to reduce the measurement of the conditions of energy transfer inside the waveguide to the measurement of effects accompanying the transfer outside it with guarantee of homogeneity of the waveguide tract.

In the article we consider questions of the rigorous solution of the problem according to the determination of the temperature on the absorbing wall, and also the time parameters, enabling us to carry out optimal designing of probes of the transmitted microwave power.

On the example of the principal type of water in a rectangular waveguide we find the dissipative function for an absorbing wall, and account of the scattering of an electromagnetic wave enabled us to form the thermal-conductivity equation with inhomogeneous boundary conditions, which physically signify the equality of the temperatures over the perimeter of the absorbing wall and the waveguide and the convection from the lateral boundaries of the absorbing wall. The solution of the thermal-conductivity equation is sought in the form of the sum of a stationary and a nonstationary temperature field by the classical Fourier method in terms of operators.

As a result of the rigorous solution we obtain that the stationary field is represented in the form of alternating series, investigated on an M-20 digital computer, and the nonstationary field is represented in the form of a collection of damped thermal waves. The curves of the stationary and nonstationary temperatures on the outer surface of the absorbing wall agree well with experiment. We present the time constant, the sensitivity, and a suitable length of the absorbing wall, which are necessary for development of optimal probes for measuring the transmitted microwave power.

SOLIDIFICATION OF AN INFINITE PLATE IN A MOLD†

É. A. Iodko and D. M. Maksimenko

UDC 669-154:620.746.6.001.24

A method is presented for a numerical solution of the problem of the solidification of a flat ingot in the mold based on a combination of an analytical solution of the inverse problem of solidification [1] and a finite-difference approximation of the heat conduction equation for the wall of the mold. In this case it is possible to realize a system of calculation which is stable even in the absence of solidification of the initial moment of a layer when the usual methods of calculation are unstable [2].

The solution of the inverse problem of solidification, written in the form of a series [1]

$$t_1 = \sum_{j=0}^{\infty} \frac{A_j(\mathrm{Fo})}{j!} (\eta - \xi)^j, \qquad (1)$$

*Khar'kov Institute of Radio Electronics. Original article submitted September 10, 1970; abstract submitted January 19, 1971.

†Original article submitted March 25, 1970; abstract submitted October 15, 1970.

where η is the coordinate, Fo is the time, and ξ(Fo) is the coordinate of the solidification front, is represented in the form

$$t_1 = \sum_{j=0}^{\infty} D_j \nu^j,$$

where

$$D_0 = 1;\ D_1 = -\frac{R}{2} \cdot \frac{dS^2}{d\,\mathrm{Fo}} - q_2 S;$$

$$D_{j+2} = \frac{1}{(j+1)(j+2)} \left\{ S^2 \frac{dD_j}{d\,\mathrm{Fo}} + \frac{1}{2} \left[(j+1) D_{j+1} - j D_j \right] \frac{dS^2}{d\,\mathrm{Fo}} \right\}.$$

Here $S = 1 - \xi$ and $\nu = (\eta - \xi)/S$. An explicit finite-difference approximation is used to calculate the temperature t_2 at the wall of the mold.

A calculating program was run on a Minsk-22 computer in accordance with this system and the kinetics were calculated for the solidification of an infinite plate in contact with walls having different thermophysical properties and thickness, among them for cast iron and fireclay walls, for a filling of coke–fireclay mixture, and for a wall made of thermally insulating tile.

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LINEARIZATION OF A NONSTATIONARY HEAT-TRANSFER PROBLEM BY A PERTURBATION TECHNIQUE

V. L. Chumakov

UDC 536.24

A procedure for the approximate solution of the nonstationary heat-conduction equation with nonlinear boundary conditions is illustrated by treating the problem of the heating (cooling) of solids by radiation and convection simultaneously for a constant ambient temperature. The gist of the method is the following.

For the boundary condition

$$(\mathrm{grad}\,\Theta)_{\Pi} = \mathrm{Bi}\,(1 - \Theta_{\Pi}) + \mathrm{Sk}\,(1 - \Theta_{\Pi}^4) \tag{1}$$

the parametric function $\Phi(\Theta_{\Pi}, q)$ is introduced with the linearization parameter q and the perturbation parameter ε:

$$(\mathrm{grad}\,\Theta)_{\Pi} = \mathrm{Bi}_{\Sigma}(q)\,[1 + \varepsilon\Phi(\Theta_{\Pi}, q) - \Theta_{\Pi}], \tag{2}$$

where the equivalent relative heat-transfer coefficient is

$$\mathrm{Bi}_{\Sigma} = \mathrm{Sk}\,(1 + p + q),\ p = \mathrm{Bi}/\mathrm{Sk}.$$

The parameter ε measures the size of the "perturbing" nonlinear complex Φ

$$\Phi(\Theta_{\Pi}, q) = \frac{\Theta_{\Pi} - \Theta_{\Pi}^4 - q\,(1 - \Theta_{\Pi})}{1 + p + q}$$

in comparison with unity. For $\varepsilon = 1$, condition (2) is identical with (1), and for $\varepsilon = 0$ we have the "unperturbed" linear condition in the form of a boundary condition of the third kind. The parameter q should be

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Original article submitted December 17, 1970; abstract submitted March 25, 1971.

chosen in such a way as to minimize the effect of the function Φ on the solution of the problem. It is convenient to take q as the mean value of the polynomial $(\Theta + \Theta^2 + \Theta^3)$.

The first step (zero approximation) in the solution of the original problem is to find Θ_0, the solution of the linearized problem ($\varepsilon = 0$). Subsequent approximations Θ_m ($m = 1, 2, \ldots$) are constructed by the small parameter method.

In the recurrent sequence of similar linear boundary value problems for Θ_m the expressions for the parametric functions

$$\Phi_m = \Phi[\Theta_{\Pi 0}, \Theta_{\Pi 1}, \ldots, \Theta_{\Pi_{m-1}}] = \frac{1}{(m-1)!}\left(\frac{\partial^{m-1}\Phi}{\partial\varepsilon^{m-1}}\right)\Bigg|_{\substack{\Theta_\Pi = \Theta_{\Pi 0} \\ \varepsilon=0}}$$

are considered as variable ambient temperatures which permit the solution for Θ_m ($m = 1, 2, \ldots$) by Duhamel's theorem.

Comparison of reference data for a wide range of Sk values with numerical solutions shows that the proposed method is accurate enough for engineering purposes. In all the examples considered the error in the first approximation does not exceed 1-1.5%. The presence of a convective heat flux ($p > 0$) increases the accuracy of the determination of the temperature distribution.

The generality of the nonlinear problem to which the proposed method is applicable is limited by the requirement that the linearized form of the problem have a solution.

SOLUTION OF GENERAL NONLINEAR PROBLEM OF UNSTEADY HEAT CONDUCTION BY SMALL-PARAMETER METHOD

L. A. Kozdoba and V. L. Chumakov

UDC 536.21

In the general nonlinear problem the dependence of the thermophysical characteristics of materials on the temperature of the internal and external heat sources is taken into account. Three different nonlinearities can be taken into account, even if they are considerable, provided that one adopts suitably selected perturbation functions for (φ, ψ).

The solution of the nonlinear problem is sought in the form of a function $f[\theta_m(\mathrm{Fo}), \theta]$ which reproduces the nonlinearity of the boundary condition (θ is dimensionless temperature; m denotes the medium). The heat-conduction coefficient (L) and the volume specific heat (C) are given by

$$C(\theta) = [a_1 + \varepsilon\varphi(\theta)]\frac{\partial f}{\partial\theta}, \quad L(\theta) = [a_2 + \varepsilon\psi(\theta)]\frac{\partial f}{\partial\theta}, \tag{1}$$

where a_1, a_2 are approximation coefficients which can be determined by using the conservation condition of the weighted-mean values of L and C within a specified range of temperatures; ε is the perturbation parameter ($0 \leq \varepsilon \leq 1$).

If $\varepsilon = 1$ the expressions (1) take fully into account the actual laws for $L(\theta)$, $C(\theta)$. In the complete article the small-parameter method was applied in the case in which the solution is known in the first approximation either of another nonlinear or of a linear equation with variable or constant coefficients for the unsteady problem. Compared with numerical solution an example of heating a plate by convection and

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Original article submitted April 24, 1970; abstract submitted May 10, 1971.

radiation simultaneously has shown that after only two approximations the maximal error does not exceed 2%. The thermal characteristics of the material and the nonlinear boundary condition depend essentially on temperature; it was, therefore, not possible to apply the standard small-parameter procedure.

APPLICATION OF THE SMALL-PARAMETER METHOD TO THE SOLUTION OF THE HEAT-CONDUCTION PROBLEM WITH TEMPERATURE-DEPENDENT HEAT SOURCES

L. A. Kozdoba and V. L. Chumakov

UDC 536.21

A procedure was given for solving the nonlinear problem of unsteady heat conduction in the case in which the heat sources were distributed uniformly by volume and their specific thermal capacity (w) was a function of time and depended strongly on the temperature.

One represents w in the form of a product,

$$w[\tau,\ T(x,\ \tau)]=\beta(\tau)\,W(T), \tag{1}$$

where W(T) is approximated within the given temperature range by a first-degree polynomial

$$P_1(T)=W_0+W_1T. \tag{2}$$

The coefficients W_0, W_1 can be determined by using any method of function approximation (say, the method of least squares).

If w is written in the form

$$w=\beta(\tau)\,[P_1(T)+\varepsilon\varphi(T)], \tag{3}$$

then φ(T) is the perturbation function and ε is the perturbation parameter ($0 \leq \varepsilon \leq 1$). For $\varepsilon = 1$ the expression (2) reproduces exactly the given w and for $\varepsilon = 0$ the function w is approximated by the linear function (2).

Subsequently, the standard procedure of the small-parameter method is applied; then the solution in the zero approximation is in fact the solution of a linear problem with a uniformly distributed source whose thermal capacity depends on temperature and also in an arbitrary manner on time.

The preliminary approximation w(T) enables one to employ the small-parameter method to solve nonlinear problems with an essential nonlinearity in w(T).

To illustrate our considerations an example was provided of heating a plate by the Joule effect with constant voltage of the voltage source,

$$w(T)=\frac{\beta}{1+\alpha(T-T_i)},$$

where α, β, T_i are given constants.

A comparison with the numerical solution has shown that the error of the solution with two approximations does not exceed 1-2%. A linear approximation of w(T) or a linearization of w by means of its mean-integral value results in errors of 11 and 32% respectively.

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Original article submitted March 23, 1970; abstract submitted May 10, 1971.

WORK OF STIRRING A LIQUID WITH GAS BUBBLES*

É. S. Geskin

UDC 532.529.5

The rate of heat and mass transfer during the ascent of gas bubbles through a liquid can be evaluated by the displacement work, i.e., the work of stirring the liquid. The work put in for generating a gas bubble and then expanding it during injection is expended in overcoming the atmospheric pressure, in displacing the liquid, i.e., in increasing its own potential energy, and in increasing the kinetic energy of the liquid. The last two components of the work will be lost on stirring the liquid. The increment of potential energy is equal to the work of Archimedes forces, which determine the ascent of a bubble, and this energy is also lost on stirring. The total work of stirring A in an isothermal system is determined by the relation

$$A = 2GRT \ln \frac{P_0 + h\gamma}{P_0} + W,$$

where G is the gas mass, T is the system temperature, h is the depth at which a cavity is formed, γ is the specific gravity of the liquid, P_0 is the atmospheric pressure, R is the gas constant, and W is the increment of kinetic energy in the liquid due to expansion of the gas.

The derived formula is a refinement of already known formulas which are based either on the work of expansion or on the work of Archimedes forces alone and which, therefore, yield results too low by one half.

The value of A can be used for estimating the rate of heat and mass transfer in the liquid during the ascent of bubbles, which is very important in the evaluation of many technologies.

As an illustration of such estimates, a comparison is made between the stirring rate with an associated gas and with an unassociated gas in a liquid; furthermore, the interaction between a gaseous oxidizer jet and the hearth of a steelmaking furnace is also analyzed. The total work of stirring the hearth with such a jet is much greater than the initial kinetic energy of the jet and, therefore, most of the work of stirring comes from the thermal energy of the oxidation reaction.

OPTIMUM CONTROL OF COOLING OF A HOLLOW CYLINDER†

D. V. Mal'tsev

UDC 536.24

A hollow cylinder of inner radius l and outer radius l is examined in the article. The cylinder is under the influence of an axially symmetrical temperature field T(r, t) which does not vary in the circumferential and axial directions but is an arbitrary function of the radius r and the time t.

The cylinder has a null initial temperature and at the time t = 0 heat exchange begins at the inner and outer cylindrical surfaces with the media which have the temperatures $T_1(t)$ and $T_2(t)$, respectively.

The end sections of the cylinder are fastened securely. The radial stresses are assumed to be equal to zero at its inner and outer surfaces.

Deformation of the cylinder can be controlled by creating a suitable program of cooling the outer cylindrical surface.

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The radial component of the deformation is designated as $\varepsilon_{rr}(r, t)$, while we take as the safe possible deformation in this direction $\varepsilon^*_{rr} = \text{const}$, so that $\varepsilon_{rr}(r, t) \leq \varepsilon^*_{rr}$. The program of cooling (control function) $T_2(t)$ must be determined in such a way that the functional

$$J = \int_1^l \left[\varepsilon^*_{rr} - \varepsilon_{rr}(r, t)\right]^2 r dr,$$

which characterizes the deviation of the deformation across the thickness of the cylinder from the given ε^*_{rr}, is minimum by the time t_{op}.

THEOREM. In order that the functional J reach a minimum for the control $T_2 = T_2^*$ and the function $\varepsilon_{rr}(r, \text{Fo})$ corresponding to it, it is necessary that the function $T_2(\text{Fo})$ satisfy the equation

$$\int_1^l |\varepsilon^*_{rr} - \varepsilon_{rr}(r, \text{Fo})| \, Q_2(r) \, r dr = 0, \tag{1}$$

$T_2(\text{Fo})$ being an arbitrary discrete-continuous function.

As a result of the solution of the stated problem it is shown that the most favorable conditions for a cylinder will be conditions where the outer surface of the cylinder is cooled according to the function

$$T_2(\text{Fo}) = [A\,\text{Fo} + T_2(0)] \exp\left[-a\gamma_1^2 \text{Fo}\right]. \tag{2}$$

The constant A is determined from Eq. (1) upon substitution of (2) into it and depends on the geometrical and thermophysical parameters of the cylinder, and on the initial temperature and thermal state of both the inner and outer cooled surfaces.

IMPURITY-ATOM DISTRIBUTION IN A SEMICONDUCTOR DURING DIFFUSION INTO A SLIT IN A MASKING FILM

Yu. F. Blinov and D. A. Sechenov

UDC 539.219.3:539.293.001.24

One of the major stages in the fabrication of semiconductor devices and integrated circuits is local diffusion of impurities into the semiconductor through a window in a protective mask on its surface.

This article considers the impurity-atom distribution in a semiconductor during diffusion from a constant source into a window in a masking film. Since the impurity-atom distribution in the center of the slit is described with high accuracy by the supplemental error function when the slit is sufficiently wide, this makes it possible to consider the solution to the composite boundary problem at the semiplane for the diffusion equation.

Use of the Viner–Kopf method permits the composite boundary conditions at the semiconductor surface to be reduced to homogeneous conditions of type I or II. Using the Laplace transform and the Fourier sine and cosine transforms, a solution to the problem at hand is then found in explicit form.

Analysis of the expressions obtained shows that they are valid when the slit width is $W \geq 8\sqrt{Dt}$, where D is the diffusion constant and t is the time. This article gives graphs representing the impurity-atom distribution functions both in the vicinity of the slit and in the region under the masking film. The results obtained can be used for calculation of the electrophysical characteristics of diffusion layers and elements of integrated semiconductor circuits, as well as for solution of a number of thermal-conductivity problems.

Taganrog Radiotechnical Institute. Original article submitted October 26, 1970; abstract submitted February 10, 1971.

IMPURITY-ATOM DISTRIBUTION DURING LOCAL DIFFUSION FROM A CONSTANT SOURCE INTO A NARROW SLIT*

Yu. F. Blinov and D. A. Sechenov

UDC 539.219.3:539.293.001.24

This article considers the impurity-atom distribution in a semiconductor during local diffusion from a constant source into a narrow slit. The slit is considered narrow when its width is $W < 8\sqrt{Dt}$, where D is the diffusion constant and t is the time.

Analytic solution of the two-dimensional diffusion equation when the diffusant concentration in the slit region is given and the amount of impurity outside the diffusant stream is zero (a masking film on the surface of the semiconductor) involves considerable mathematical difficulties.

Analysis of the characteristics of the impurity distribution near the edge of the masking film with a slit width $W \gg 8\sqrt{Dt}$ made it possible to reduce the composite boundary problem to a homogeneous problem with approximate boundary conditions of type I, for which an analytic solution was found with the Laplace transform and the Fourier sine transform. The diffusion-equation solutions obtained were shown to be suitable for description of the impurity-atom distribution during diffusion from a constant source into a slit of width $W \geq 0.4\sqrt{Dt}$. At smaller slit widths, the impurity distribution in the semiconductor corresponds to diffusion from a cylindrical source located at the surface of the semiconductor.

The results obtained can be used for calculation of the electrophysical characteristics of diffusion layers and elements of integrated semiconductor circuits.

EFFECT OF TEMPERATURE PROFILE IN A PERMEABLE THERMOELECTRIC REFRIGERATING BATTERY ON ITS ENERGETIC CHARACTERISTICS†

G. K. Kotyrlo and G. M. Shchegolev

UDC 536.12

The principal efforts in improving the efficiency of thermoelectric devices are directed toward the discovery of new thermoelectric materials and toward improving the technology of manufacturing thermoelements.

A new kind of effect on the energetic efficiency of thermoelectric devices is examined in the article. Its essence consists in the fact that the supply (or removal) of the bulk of the heat to the material of the thermoelement branches is accomplished not through the surface of the junctions but within the branches, which are made permeable to the cooling substance or heat carrier (coolant) in the case when the thermoelement is operated as a generator of energy.

Because the inner heat-exchange surface of permeable thermoelements can be extremely developed,

*Taganrog Radiotechnical Institute. Original article submitted October 26, 1970; abstract submitted February 10, 1971.

† Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Original article submitted December 9, 1969; abstract submitted May 25, 1971.

heat exchange between the circulating liquid (gas) and the solid material takes place at small temperature differences, i.e., almost reversibly.

Having examined the temperature curves in the material of monolithic and permeable branches, the properties of such thermoelements are easy to represent. The temperature profiles in both cases have a nonlinear nature. The temperature gradients at the junctions, and therefore the amount of heat leaving the hot junctions and entering the cold junctions as a result of the thermal conduction of the material of the branches, will differ.

In the case of monolithic thermoelements the distortion of the temperature curve is caused by the liberation of Joule heat, and the temperature gradients increase in the direction from the hot junctions to the cold. In permeable thermoelements, owing to the supply of heat from the cooling substance to the material of the branches, the temperature curve is even more distorted within the volume of the thermoelement and the amount of heat entering the cold junctions due to thermal conduction is increased in comparison with monolithic thermoelements. Having written the expression for the cooling coefficient in the form

$$\varepsilon = \frac{Q_{ph} - \frac{\lambda}{\operatorname{tg}\alpha} - W}{W},$$

one can see that the greater the angle α (the angle formed by the tangent to the temperature curve and the axis normal to the surface of the hot junction), the greater the cooling coefficient. In this expression Q_{ph} is the Peltier heat liberated at the hot junctions and W is the applied power. Thus, a ventilated cooling thermoelement will be more economical than one which is not ventilated, other things being equal.

In order to conduct a quantitative analysis of the operation of permeable thermoelements a method was developed for their calculation, on the basis of which it is assumed that the cooling substance enters the thermoelement from the side of the hot junctions, the temperature of which is maintained equal to the initial temperature of the coolant. The removal of heat from the coolant takes place within the thermoelements, while the Peltier effect at the cold junctions is compensated for only by the heat arriving there by means of the thermal conduction of the material.

The calculations which were conducted for a permable thermoelement and its comparison with a monolithic thermoelement showed the advantages of a ventilated thermoelectric refrigerator with respect to energetic indices in application to several cooling systems.

EFFECT OF RADIATION AND OF A TEMPERATURE JUMP ON THE COOLING OF A THIN WIRE IN A GAS LAYER

A. S. Umanskii and Yu. A. Gorshkov

UDC 536.12:23

The article deals with the transient problem of cooling in the case of a thin rod (wire) placed coaxially inside a gas-filled cylinder; the boundary conditions stipulate a temperature jump at the rod surface (a so-called Knudsen jump) and a radiative heat transfer from the rod to the cylinder wall.

An exact solution to the problem is obtained and the effects of the temperature jump at the gas–wall boundary as well as of the radiation on the characteristic cooling time of the system are analyzed.

Approximate analytical expressions are obtained for the characteristic times of the system (within an accuracy down to the terms of second-order smallness):

$$x_i = x_i^0 + \Delta_{rad}^{(i)} + \Delta_{ju}^{(i)}.$$

Institute of High Temperatures, Academy of Sciences of the USSR, Moscow. Original article submitted September 16, 1970; abstract submitted June 14, 1971.

Here $x_i^0 \simeq (3/4 + i)\pi[1-(\pi^2/25)(3 + 4i)(C_1/C_2-1)y^2]$; i = 1, 2, . . . are the eigenvalues of the problem without consideration of the radiation and the temperature jump;

$$\Delta_{\text{rad}}^{(1)} \simeq \frac{8\sigma\varepsilon T_0^3}{\lambda_2} R_1 \ln\frac{8R_2}{3\gamma\pi R_1};$$

$$\Delta_{\text{ju}}^{(1)} = -\frac{3^4}{4^5}\pi^4 K_{\text{ju}}\left[R_1\frac{C_1}{C_2}\right]^2\frac{1}{R_2^3}\ln\frac{8R_2}{3\gamma\pi R_1},$$

where T_0 is the absolute temperature of the outer surface of the system, R_1 and R_2 are the radii of the rod and of the cylinder respectively, C_1/C_2 is the ratio of the volume specific heats of the rod material and the gas, and K_{ju} is a constant which determines the magnitude of the Knudsen temperature jump and which depends on the coefficient of thermal accomodation as well as on the thermophysical parameters of the system – it is defined as for steady-state conditions. The correction for the temperature jump contains, besides the parameters which determine the magnitude of the steady-state temperature jump, also the ratio of volume specific heats of the rod material and the gas.

RATE OF BLASTING ON A SOLID PARTICLE IN A PULSATING GAS STREAM

V. S. Severyanin and S. G. Ushakov

UDC 533.6:662.61

One method of intensifying the burning of a solid or a liquid fuel is by the use of so-called pulsating-combustion chambers. As of now, however, the aerodynamic process conditions under which fuel particles burn in a pulsating stream are still not quite well understood.

The article presents an analysis of the equation of motion for a spherical solid particle not subjected to external forces in a one-dimensional gas stream going through forced sinusoidal pulsations, and a solution to this equation obtained with the aid of a digital computer is shown. Among the dimensionless parameters varied here over wide ranges are not only the physical particle and gas constants but also the referred particle diameter as well as the amplitude of the stream velocity and the frequency of pulsations. Also the adequate parameters, namely the initial particle velocity and the constant component of the stream velocity, are analyzed by a variation of the initial conditions.

On the basis of the calculated formula for the dimensionless rate of blasting on a particle as a function of time the amplitude–phase characteristics of particle vibrations are plotted versus the governing parameters.

Even for sufficiently fine particles (30-50 μ) within the range of moderate acoustic frequencies characteristic of pulsating-combustion chambers (180-300 Hz), the pulsations of the stream velocity are utilized 70-80% when the continuous blast on the particles is negligible. The phase shift (during steady-state pulsations of the particles) is determined uniquely by the gain parameter $B = 3\nu\rho/4\omega\rho_2\delta^2$, increasing as the particle size δ and the gas pulsation frequency ω decrease (ν and ρ_1 are the kinematic viscosity and the density of the gas respectively; ρ_2 is the particle density). On the basis of the calculated results, the following formula is proposed for determining the effective rate of blasting on a particle with a pulsating stream in a process of duration τ:

$$w_{\text{eff}} = w\frac{\pi}{\omega\lambda\tau} + \frac{v_0 z}{\sqrt{2}},$$

where $w = w_0 + v_{av}$; w_0 is the initial velocity of a particle, v_{av} is the constant component of the gas velocity, λ is the damping coefficient for the initial velocity (characterizing the duration of the transient), v_0 is the amplitude of the gas velocity, and z is the relative amplitude of the blast rate.

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Original article submitted February 19, 1971; abstract submitted July 16, 1971.

ON THE ROLE OF ADSORBED GAS DURING BOILING

Yu. G. Ershov and V. I. Khrenov

UDC 536.423.1:66.046.7

For the purpose of studying the effect of adsorbed gas on the boiling process, experiments were performed in filling cavities held in a liquid for a long time and after a short boiling period. Conical capillaries fused at one end, 0.24-10.0 mm long and with an orifice diameter 0.004-0.18 mm, were used as test models. The tests were performed with distilled water, transformer oil, and 20% NaCl solution. It is to be noted that this range of capillary orifice diameters covers the actual cavities on metallic surfaces where heat transfer occurs.

An analysis of the experimental data leads to the following conclusions:

1) as the diameter of a capillary becomes smaller, the capillary fills up faster (the relative fill height h/d increases);

2) the smaller the surface tension of the liquid and, thus, the better the wettability is, the sooner will the capillary be filled with the liquid;

3) the magnitude of the diameter affects the rate of filling particularly strongly within the range up to 50 μ;

4) the column height as well as the rate of rising are affected by the length of a capillary at a constant orifice diameter or, which is equivalent, by its conicity d/l: the obtained data have shown that the rate at which a liquid fills capillaries of the same diameter is directly proportional to their conicity, at least within the range of angles from 0.08' to 8°00'.

It ought to be mentioned that one important characteristic of conical cavities, which distinguishes them from cylindrical ones, is that the former are necessarily filled with liquid at a wetting angle $\theta = 90°$. Since in a conical cavity there can be no equilibrium between the trapped gas and the liquid, hence any displacement of the interphase boundary results in a smaller curvature radius of the meniscus and thus, in turn, an increase of the pressure above it as well as of the diffusion rate and in a further displacement of the interphase boundary. In the case of long (several centimeters) cylindrical capillaries, on the other hand, such an equilibrium between phases is possible because of the saturation of the liquid in a capillary with air.

All this is valid for liquids which wet a solid surface, i.e., form with it an angle $\theta < 90°$. Surface – liquid combinations with an angle $\theta > 90°$ are extremely rare in practice. One may hypothesize that there exist no solid surfaces which are not wettable by a liquid (at least by a boiling liquid), because such surfaces are to some extent oxidized by the liquid (produce a chemical compound with it) and oxidized surfaces are wettable. The difference, evidently, lies only in the degree of wettability, i.e., in the rate at which an adsorbed gas is removed by diffusion, by dissolution, or by the formation of surface oxides in the presence of that gas.

In this way, an adsorbed gas trapped in a cavity by a liquid at the heating surface can only at the very beginning serve as a nucleus of vapor formation during boiling. Since this gas can easily be removed by various methods (artificial cavitation, holding the heat-transfer surface in the liquid for a long time, subjecting the solid surface to high pressure, or, finally, brief boiling) and, therefore, one may assume that the main source of vapor formation on a solid surface is the appearance of a gas phase when this surface interacts with the liquid.

V. I. Lenin Power Institute, Ivanovo. Original article submitted October 26, 1970; abstract submitted June 15, 1971.

CALCULATING THE LUMINOSITY CHARACTERISTICS OF LIGHT SCATTERING MEDIA. I

K. S. Adzerikho and V. P. Nekrasov

UDC 536.3:536.52

The article deals with an analysis of the effects which the optical properties of a layer (optical thickness, quantum survival probability, dispersion indicatrices, size distribution of particles) and which the boundary conditions have on the luminosity of test objects when this luminosity is calculated by various methods. A comparison of the numerical results obtained for the emissivity of a plane layer by various methods of calculation is useful in establishing the applicability range of each method and in choosing the most suitable one for any given practical case.

When interpreting experimental data in terms of a certain relation involving the emissivity of a light scattering layer, it is common to introduce a number of semiempirical corrections. This is, evidently, because the effect of one or another parameter is not accounted for when the luminosity of a light scattering medium is analyzed. In this article the problem concerning the luminosity of light scattering media is treated rigorously in the Schwarzschild–Schuster approximation, considering the optical characteristics of the medium as well as the boundary conditions. The following expression is taken as the indicatrix of the radiation scatter into a volume element:

$$p(\mu, \mu') = a + 2(1 - a)\delta(\mu - \mu'),$$

where δ is the delta function, $\alpha = 2\beta$, β is the hemispherical backscatter factor, and $\mu = \cos\theta$. Such a representation describes qualitatively the process of radiation scatter at particles whose size is comparable to or larger than the wavelength, while, at the same time, it reduces the original equation to an equation of radiation transmission with a spherical radiation indicatrix. For optical thicknesses $\tau_0 \geq 5$ the emissivity of a light scattering layer depends considerably on the parameter β, which determines the elongation of the scatter indicatrix. A study of this relation makes it possible to establish the characteristic dimensions of particles in a medium at known quantum survival probabilities and radiation wavelengths.

The solution, in the Schwarzschild–Schuster approximation, to the equation of radiation transmission through a medium with uniformly distributed radiation sources is used here for determining the angular distribution of luminosity in the layer. In the case of a semi-infinite medium, the angular distribution of emissivity is defined rather simply by the following expression:

$$\varepsilon(\mu) = (1 + 2\mu)(k + 2\mu)^{-1},$$

where $k = [1 + 2\beta\lambda/(1-\lambda)]^{1/2}$ and $\lambda = \sigma/(\varkappa + \sigma)$ is the quantum survival probability. A comparison of specific calculations according to this formula with the exact solution to the equation of radiation transmission indicates that this formula is very accurate. Its validity for the case of small optical thicknesses is demonstrated in Part II of this article.

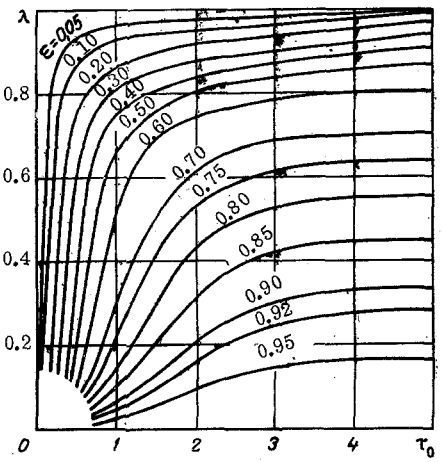

Fig. 1. Nomogram for determining the hemispherical emissivity of a finite light scattering layer.

A thorough analysis of the emissivity of a light scattering layer, as a function of the optical properties of the medium, has made it possible to construct a nomogram convenient for practical determinations of the hemispherical emissivity for layers of finite optical thickness (Fig. 1). It is worthwhile to use this nomogram for determining the spectral emissivity characteristic of a layer and, thus to reduce considerably the amount of computation effort.

Institute of Physics, Academy of Sciences of the Belorussian SSR, Minsk. Original article submitted July 7, 1970; abstract submitted January 27, 1971.

CALCULATING THE LUMINOSITY CHARACTERISTICS OF LIGHT SCATTERING MEDIA. II*

K. S. Adzerikho and V. P. Nekrasov

UDC 536.3:536.52

In order to confirm the validity of the results obtained in the first part of this article, the solution to the equation of radiation transmission is analyzed for the case of a medium with uniformly distributed sources and considering scatter by single reflections. A comparison of numerical results according to the formula obtained here at $\tau_0 = 0.05$ with those based on relations in Part I indicates that the latter are valid. It appears, furthermore, that, within a 5% error, the normal emissivity under single reflections may be calculated up to $l = 2\beta\lambda/(1-\lambda+2\beta\lambda) = 0.6$ inclusive (β is the hemispherical backscatter factor and λ is the quantum survival probability). As λ decreases, the validity range of calculations based on single reflections widens.

The problem concerning the luminosity of a light scattering medium of finite optical thickness is also solved by the method of spherical harmonics in the P_1-th and P_3-th approximation. It is shown that for determining the constants in the solution one must use boundary conditions of the Marshak kind. It is demonstrated by way of specific calculations that in the P_1-th approximation only the hemispherical emissivity can be calculated with a sufficiently high accuracy. The P_3-th approximation improves the accuracy to almost twice as close (see Table 1). As in the P_1-th approximation, the accuracy of calculations increases as the quantum survival probability becomes higher. The calculation of the normal emissivity of a layer in the P_1-th and the P_3-th approximation yields a 20-25% error, which indicates how slowly convergent the method of spherical harmonics is when used for calculating the angular distribution of radiation leaving a layer. Among the decisive advantages of the method of spherical harmonics is the feasibility of using it when the geometry of a medium is nonplanar.

TABLE 1. Values of the Hemispherical Emissivity for a Semiinfinite Layer in the P_3-th Approximation

l	0,4	0,5	0,6	0,7	0,8	0,9
ε	0,847	0,805	0,754	0,690	0,605	0,471
Δ (%)	3,1	2,8	2,7	2,4	2,1	1,9

STRESS FIELD IN A WEDGE-SHAPED MASS WITH A CONCENTRATED INTERNAL FORCE†

R. V. Tedeev

UDC 532.12

The article deals with the action of a vertical force (P) and a horizontal force (Q) concentrated inside an infinitely long wedge with an arbitrary angle and in a state of two-dimensional strain (Fig. 1). In polar coordinates the stress components (radial σ_r, tangential σ_θ, and shearing $\tau_{r\theta}$) and the displacement components (radial ξ and tangential η) are expressed as follows:

$$\sigma_r = \frac{1}{r}\cdot\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2\Phi}{\partial\theta^2};\quad \sigma_\theta = \frac{\partial^2\Phi}{\partial r^2};\quad \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\cdot\frac{\partial\Phi}{\partial\theta}\right);$$

*Institute of Physics, Academy of Sciences of the Belorussian SSR, Minsk. Original article submitted July 30, 1970; abstract submitted January 27, 1971.

†Institute of Geophysics, Academy of Sciences of the USSR, Moscow. Original article submitted December 22, 1970; abstract submitted August 25, 1971.

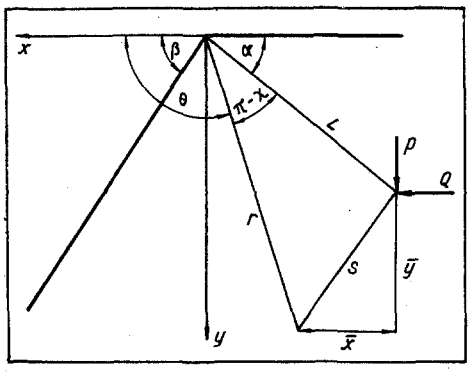

Fig. 1

$$\frac{\partial \xi}{\partial r} = \frac{1}{2G}[(1-\nu)\tau_{\sigma_r} - \nu\sigma_\theta]; \quad \frac{\xi}{r} + \frac{1}{r}\cdot\frac{\partial \eta}{\partial \theta} = \frac{1}{2G}[-\nu\tau_{\sigma_r} + (1-\nu)\sigma_\theta];$$
$$\frac{1}{r}\cdot\frac{\partial \xi}{\partial \theta} + \frac{\partial \eta}{\partial r} - \frac{\eta}{r} = \frac{1}{G}\tau_{r\theta}, \tag{1}$$

where G is the shear modulus, ν is the Poisson ratio, and Φ is an arbitrary biharmonic function.

In order to solve the problem, the biharmonic function is put in the form

$$\Phi = (\Phi^k - \Phi^0) + \Phi^*. \tag{2}$$

Here the function

$$\Phi^k - \Phi^0 = r^2 \int_0^\infty [\cos m\rho\,(a^k \operatorname{ch} m\chi + B^k \operatorname{sh} m\chi + c^k \operatorname{ch} m\chi \cos 2\chi + C^k \operatorname{sh} m\chi$$
$$\times \sin 2\chi - d^k \operatorname{ch} m\chi \sin 2\chi + D^k \operatorname{sh} m\chi \cos 2\chi) + \sin m\rho\,(b^k \operatorname{sh} m\chi - A^k \operatorname{ch} m\chi$$
$$+ c^k \operatorname{sh} m\chi \sin 2\chi - C^k \operatorname{ch} m\chi \cos 2\chi + d^k \operatorname{sh} m\chi \cos 2\chi + D^k \operatorname{ch} m\chi \sin 2\chi)]\,dm \tag{3}$$

is the Kelvin solution for a force applied within an infinite plane, while the function

$$\Phi^* = r^2 \int_0^\infty [\cos m\rho\,(a^* \operatorname{ch} m\delta + B^* \operatorname{sh} m\delta + c^* \operatorname{ch} m\delta \cos 2\delta + C^* \operatorname{sh} m\delta$$
$$\times \sin 2\delta - d^* \operatorname{ch} m\delta \sin 2\delta + D^* \operatorname{sh} m\delta \cos 2\delta) + \sin m\rho\,(b^* \operatorname{sh} m\delta - A^* \operatorname{ch} m\delta$$
$$+ c^* \operatorname{sh} m\delta \sin 2\delta - C^* \operatorname{ch} m\delta \cos 2\delta + d^* \operatorname{sh} m\delta \cos 2\delta + D^* \operatorname{ch} m\delta \sin 2\delta)]\,dm \tag{4}$$

represents another biharmonic function added to the Kelvin solution and extends the solution to a wedge. The following designations have been made in (3) and (4):

$$\rho = \ln\frac{r}{L}; \quad \chi = \theta + \alpha; \ (-\pi \leqslant \chi \leqslant \pi); \quad \delta = \theta + \varepsilon,$$

where ε is a nonessential constant arbitrarily chosen; for convenience, we let $\varepsilon = -(\pi + \beta)/2$.

Inserting the value of the biharmonic function Φ into (1) and performing the necessary operations, we obtain expressions for the stresses and the displacement components. The arbitrary constants a^k, A^k, b^k, B^k, c^k, C^k, d^k, D^k are found from the Kelvin solution, but the arbitrary constant a^*, A^*, b^*, B^*, c^*, C^*, d^*, D^k are found from the boundary conditions at the wedge planes. The case is analyzed where the wedge planes are free of stress, i.e., $\sigma_\theta = 0$ and $\tau_{r\theta} = 0$ at $\theta = \beta$ and $\theta = \pi$. Equating separately to zero the cos mρ terms and the sin mρ terms in the integrands in the expressions for the stresses σ_θ and $\tau_{r\theta}$ at $\theta = \beta$ and $\theta = \pi$ we construct a system of eight equations, the solution of which yields the unknown arbitrary constants.

According to the final stress and displacement formulas including the arbitrary constants, the stresses vanish at infinity. The solution based on the biharmonic function Φ^* is convergent for all points inside the wedge. The convergence becomes slower as the point of force application approaches a wedgw plane. In order to improve the convergence then, one must subtract the asymptotic values at m $\to \infty$ from the integrands in the formulas for stresses and displacements, and then add them after they have been expressed in terms of simplest functions.

A CRITERION FOR EVALUATING THE EFFECT OF RADIATION ON THE TOTAL HEAT TRANSMISSION IN SEMITRANSLUCENT MEDIA

A. A. Men' and O. A. Sergeev

UDC 536.24

An analysis is made of methods published in the technical literature which are used for estimating the effect of radiation on the heat transmission in materials where radiation is a secondary mechanism of energy transfer (semitranslucent media). It is shown that the phenomenological theory, which is being developed now, hinges on a stringent quantitative criterion for comparing various approximate models with one another and with the exact solutions. The temperature drop $\Delta T = T_1 - T_2$ across a layer of semitranslucent material is proposed to serve as such a criterion. This quantity is usually found in the course of determining the temperature field with a given total (radiative and conductive) energy flux Q and a fixed temperature T_1 at one of the layer boundaries. On the basis of the criterion, a comparison is made between the approximate analytical solutions to the problem of radiative–conductive heat transmission by L. P. Filippov, by V. N. Adrianov, and by G. Poltz, and the exact computer solution of the integral equation describing the steady-state temperature distribution in a semitranslucent plate. Fused quartz has been chosen as the specific material for this study. Temperature T_1 was varied from 400 to 1500°K. The reflectivity of the layer boundaries was varied from 0 to 0.9.

Some results of the calculations are shown in Table 1. We note, for comparison, that without accounting for radiation $\Delta T = 25$°K for H_1 and $\Delta T = 50$°K for H_2 under the same conditions. Evidently, the values for ΔT obtained by those few authors are considerably at variance. For a thin layer the Poltz solution is very close to the exact one over the entire temperature range. As the layer becomes thicker, the error of all approximate solutions increases rapidly.

TABLE 1. Magnitudes of the Temperature Drops (ΔT, °K) Obtained by the Exact Numerical and by Approximate Analytical Solutions to the Problem of Radiative-Conductive Heat Transmission

Thickness, mm	H_1=5						H_2=30					
Temperature, °K	1500			1000			1500			1000		
Optical thickness	0,070			0,138			0,420			0,828		
Reflectivity of the boundaries	0	0,5	0,9	0	0,5	0,9	0	0,5	0,9	0	0,5	0,9
Exact solution (computer)	4,05	8,7	17,8	10,0	15,8	21,7	1,95	4,05	10,3	6,7	11,2	18,4
G. Poltz solution	3,9	8,4	17,4	9,8	15,5	20,8	1,78	3,10	7,8	6,4	10,0	14,4
L. P. Filippov solution (optically thin layer)	6,8	13,2	23,3	13,7	20,3	24,9	2,4	10,7	—	6,2	29,8	—
V. N. Adrianov solution	7,3	13,3	21,5	14,7	19,7	23,0	3,9	7,3	11,0	12,8	17,8	23,8
Rosseland solution	0,23	0,23	0,23	1,4	1,4	1,4	0,46	0,46	0,46	2,8	2,8	2,8

Original article submitted July 20, 1970; abstract submitted November 16, 1970.

RADIATIVE HEAT TRANSMISSION THROUGH A LAYER WITH A HIGH CONCENTRATION OF PARTICLES

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UDC 536.3

The article deals with the transmittivity and the reflectivity of a layer containing coarse gray particles. Radiation impinging on the layer surface is hemispherical. In this case, when the particles occupy a very small fraction of the total volume, $\gamma \ll 1$, the solution can be expressed by moments of the Ambartsumyan functions α_i, β_i [1] tabulated in [2]. The following expression has been obtained in [1] for the reflectivity:

$$R_0(\lambda, \tau_0) = 1 + \lambda(\alpha_0\alpha_1 - \beta_0\beta_1) - 2\alpha_1, \tag{1}$$

where λ denotes the probability of a photon surviving colllision. The dispersion indicatrix is spherical. For the transmissivity the author has derived the following expression:

$$D_0(\lambda, \tau_0) = 2\beta_1 + \lambda(\alpha_1\beta_0 - \alpha_0\beta_1). \tag{2}$$

The optical thickness of a layer τ_0 of a homogeneous medium is the ratio of the actual layer thickness to the free path length of a photon l.

If one considers that the particles at the layer surface have flat boundaries coinciding with the boundary planes of the layer, then for the mean values of R and D at any value of γ one may write:

$$R(\lambda, \gamma, \tau_0) = \lambda\gamma + (1-\gamma) R_0(\lambda, \tau_0), \tag{3}$$

$$D(\lambda, \gamma, \tau_0) = (1-\gamma) D_0(\lambda, \tau_0). \tag{4}$$

Account is taken here of the radiation reflected by the particles at the layer boundary, while the amount of radiation diffused in the interstitial space between particles is described in terms of conventional heat transmission theory. The mean free path length of photons depends on γ. Considering that the particles are randomly distributed in the medium but cannot occupy space where other particles are located, we have

$$l = \frac{1-\gamma}{n_0\sigma}, \tag{5}$$

where n_0 denotes the average number of particles per unit volume and σ denotes the median cross section of a particle. Equation (5) is proved here by the method shown in [3].

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CALCULATION OF TRAJECTORIES OF MOTION OF PARTICLES IN CURVED CHANNELS

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UDC 532.629+621.928.38

Physically the main point of the proposed calculation method consists in the fact that the trajectory of motion of a particle can be divided into infinitesimally small sections, which can be replaced by linear segments, for which the use of a differential equation of motion of the particle in vector form is valid [1]:

$$m\frac{d\mathbf{v}}{dt} = CF\frac{\rho_g}{2}(\mathbf{u}-\mathbf{v})^2.$$

To solve this equation we represent it in the form of projections on the axis of the normal coordinate system X, Y:

$$m\frac{dv_x}{dt} = \frac{\alpha v}{|\mathbf{u}-\mathbf{v}|\,d}F\frac{\rho_g}{2}|\mathbf{u}-\mathbf{v}|\,(u_x-v_x).$$

The equations will also have a similar form along the Y axis.

Cancelling the modulus of the relative velocity of particle motion $|\mathbf{u}-\mathbf{v}|$, and expanding the value of the quantities m and F, having first denoted the ratio $3\nu\rho_g/4d^2\rho_s$ by λ, we obtain a system of equations, which can be solved if the velocity components of the motion of the flow u_x and u_y are known. Taking this into account, the final solutions of the equations for the determination of the value of the projection of the velocity vector of the motion of a particle and the position coordinate of a particle on the X and Y axes will have the form

$$v_x = v_{xi} + \alpha_i\lambda t\,(u_{xi}-v_{xi}); \quad x = x_i + tv_{xi};$$
$$v_y = v_{yi} + \alpha_i\lambda t\,(u_{yi}-v_{yi}); \quad y = y_i + tv_{yi}.$$

The value for the coefficient α_i is taken from the appropriate table as a function of the Reynolds number

$$\mathrm{Re}_{di} = \frac{d}{\nu}\sqrt{(u_{xi}-v_{xi})^2+(u_{yi}-v_{yi})^2}.$$

The calculation time interval between neighboring points of the trajectory t is taken equal to or less than the particle relaxation time $\tau = d^2\rho_s/18\nu\rho_d$. In this case the inertial path of the particle corresponds to its assumed interval of motion.

The trajectories are constructed in a Cartesian coordinate system based on separate points whose coordinates correspond to the x and y values.

The calculation method is illustrated by the example of an inertial dust collector of helical type. Calculation results are compared with experiment.

The proposed simplified method of calculating trajectories can be used for curved streams with various velocity distribution laws over the cross section and a variable radius of curvature.

The regime of particle motion can vary over a rather wide range of Reynolds numbers. If it not necessary to use a computer for the numerical solution of the differential equations in the given case.

NOTATION

m, d, F	are respectively the mass, diameter, and area of the midsection of a particle;
v, u	are respectively the velocity vector of the particle and of the gas flow;
ρ_s	is the density of a solid particle;
ρ_g	is the density of the gas flow;
C	is the coefficient of aerodynamic drag of the particle;
ν	is the coefficient of kinematic viscosity of the flow;
α	is a coefficient.

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